



## AN ANALYSIS OF THE GRAPH PROPERTIES

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### ABSTRACT

In this paper we introduced Group theoretic graph  $\mathcal{E}_{2n}$ , even free graph associated with the finite abelian group  $(\mathbb{Z}_{2n}, \oplus)$  for each  $n \geq 1$ , also we explore structural properties of  $\mathcal{E}_{2n}$  and including enumeration of edges and cycles in  $\mathcal{E}_{2n}$ .

**Keywords:** Graph theory, Group theory and Combinatorics

### INTRODUCTION

Let  $G = (V, E)$  be a finite, simple, non-directed graph. The vertex set of  $G$  is described as  $V(G)$  (or simply  $V$ ) and the edge is identified as  $E(G)$  (or  $E$ ). If the edge  $e = uv$  then we state that the vertex  $u$  is adjacent to the vertex  $v$  or the vertices  $u$  and  $v$  are neighbors and that  $e$  is incident to  $u$  and  $v$ . We use the notations  $n = |V|$  and  $m = |E|$  to indicate the order and the scale of  $G$ , respectively. Subgraph  $H$  of  $G$  is a graph such that  $V(H) \subseteq V(G)$  and  $E(H) \subseteq E(G)$ . The induced sub-graph  $H$  of  $G$  (denoted by  $\langle H \rangle$ ) is a sub-graph with the added property that if  $u, v \in V(H)$ , then  $uv \in E(H)$  if and only if  $uv \in E(G)$ .  $G - \{v\}$  is considered the vertex-deleted sub-graph of  $G$  and we compose  $G - v$  instead of  $G - \{v\}$ . If  $F$  is a subset of the edges of  $G$ , then the spanning sub-graph of Graph  $G$  is a sub-graph comprising all the vertexes of  $G$  and the edges of  $F$  to  $E(G)$ . In particular, if  $F \subseteq E(G)$ , then the spanning sub-graph  $G - \{vu\}$  is called the edge-delete sub-graph of  $G$ . We are writing  $G - vu$  instead of  $G - \{vu\}$ .

If  $u$  and  $v$  are non-adjacent vertices of  $G$ , then  $G + vu$  denotes the graph with the vertex set of  $V(G)$  and the edge set of  $E(G) \cup \{vu\}$ . For graph technical notations, we obey Haynes et al. [10]. The degree of vertex  $v$  (denoted by  $\deg(v)$ ) is equivalent to the number of vertexes adjacent to  $v$ . The minimal degree (respectively the maximum degree) of graph  $G$  is denoted by  $\Delta(G)$  (respectively  $\delta(G)$ ). If there is a  $V(G)$  vertex  $v$  such that  $\deg(v) = 0$  then  $v$  is considered an independent vertex. If  $\deg(v) = 1$  then  $v$  is referred to as the end vertex or pendant vertex. The graph is assumed to be  $k$ -regular if each vertex has a degree of  $k$ . For the edge range  $F$  to  $E$ ,  $V$



(F) is the range of the end vertices of the edges of F. The direction is an alternating series of vertices and edges, such that each edge follows the vertex that precedes it and the vertex accompanies it, and no vertex is replicated. If there is a path from each vertex in Graph G to some other vertex in G then G is said to be connected, otherwise G is disconnected. The open region of the vertex v (denominated by  $N(v)$ ) is a set of vertices of G adjacent to v, i.e.  $N(v) = \{u \in V \mid uv \in E\}$ . The closed field of the vertex v is  $N[v] = N(v) \cup \{v\}$ . Enable S to have V and vertex v to have S, then the private neighborhood of v with respect to S (denominated by  $pn[v, S]$ ) is the set of vertices  $\{w \in V \mid N[w] \cap S \neq \emptyset\}$ .

The external private region of v with respect to S (denominated by  $epn(v, S)$ ) is also the set of vertices  $\{w \in V - S \mid N(w) \cap S \neq \emptyset\}$  [6]. The external private neighborhood  $epn(v, S)$  of v with respect to S consists of the private neighbors of v which are in  $V - S$ . Thus,  $epn(v, S) = pn[v, S] \cap (V - S)$ . The dominant set  $S \Delta V$  of G is a set of vertices, so that each vertex v is either in S or adjacent to the vertex of S. The dominant number (denominated by  $\Delta(G)$ ) of G is the minimum cardinality of the dominant range of G [10]. Even S is a complete dominant set if each vertex v is adjacent to the vertex of S. The absolute dominate number (denoted by  $\Delta_t(G)$ ) of G is the minimum cardinality of the absolute dominant range of G [10].

A vertex covering the range  $S \Delta V$  of G is a set of vertices such that each edge of G has at least one end vertex in S. The vertex covering the number (denoted by  $\Delta_0(G)$ ) of G is the minimum cardinality of the vertex covering the range of G [10]. A distinct set  $S \Delta V$  of G is a set of vertices such that there are no two vertices of S adjacent to it. The independence number (referred to as  $\beta_0(G)$ ) of G is the highest cardinality of the independent set of G [10]. Notice that  $\Delta(G) \leq \beta_0(G)$  for all G [10] graphs. In 1959, Gallai [7] introduced his classical theorem, which included the vertex number  $\Delta_0(G)$  and the vertex independence number  $\beta_0(G)$  as  $\Delta_0(G) + \beta_0(G) = n$ , for any graph  $G = (V, E)$  with n vertices.

The dominance, the coverage and the freedom of the graph are several fields of study.

Some research papers have been published in these areas. We have considered the edge domination and introduced a variant of edge domination in chapter 2, edge covering in chapter 4 and edge independence in chapter 5 from several viewpoints including vertex removal, edge removal and edge addition to the graph. Hyper graphs are natural extensions of graphs in which edges may consist of more than 2 vertices. The hyper graph is an order pair  $G = (V, E)$ . The nonempty set V contains the elements  $\{v_1, v_2, \dots, v_n\}$  and  $E = \{C_1, C_2, \dots, C_m\}$  is a

family of nonempty subsets of V such that  $\bigcup_{i=1}^m C_i = V$ . The elements of V and E are called the



vertices and the edges of the hyper graph  $G$  respectively. If the hyper graph  $G$  is clear from the context, we simply write  $V = V(G)$  and  $E = E(G)$ . We shall use the notations  $n = |V|$  and  $m = |E|$  to denote the order and size of the hyper graph  $G$ , respectively. If an edge  $E_1 \in E$  then  $|E_1|$  is the size of an edge  $E_1$ . An isolated edge of the hyper graph  $G$  is an edge that does not intersect any other edge of  $G$ . We make the following conventions about the hyper graph considered throughout the work unless otherwise stated.

- (1) When  $x$  and  $y$  are different vertexes, there is almost one edge comprising  $x$  and  $y$ .
- (2) Some two different edges of the hypergraph shall be intersected in at most one vertex.
- (3) If  $e$  and  $f$  are edges with at least two vertexes, then  $e \cap f \neq \emptyset$  and  $f \cap e \neq \emptyset$ . Let  $v$  be the vertex of the hyper graph  $G$  such that  $\{v\}$  is not the edge of the hyper graph  $G$ , so the sub hyper graph  $G - v$  of  $G$  is a hyper graph whose vertex set is  $V - \{v\}$  and the edge set is equal to  $\{e \in E \mid v \notin e\}$ , and  $e \cap \{v\} = \emptyset$ ,  $e \in E(G)$  is null. A partial sub-hypergraph  $G - v$  of  $G$  is a hyper-graph with a vertex set of  $V - \{v\}$  and an edge set equivalent to  $\{e \in E \mid v \notin e\}$ . Two  $x$ -and  $y$ -vertexes of the hypergraph  $G$  are adjacent if the edge  $E_1$  of  $G$  is such that  $\{x, y\} \subseteq E_1$ . The collection of vertices  $S \subseteq V(G)$  of the hyper graph  $G$  is a dominant collection of  $G$  if for each vertex  $v \in V(G) - S$  there is a vertex  $u \in S$  such that the vertices  $u$  and  $v$  are adjacent to  $G$ . A set of vertices  $S \subseteq V(G)$  is an  $H$ -dominating set of hyper graphs  $G$  if, for each vertex  $v \in V(G) - S$ , there is an edge  $F$  containing  $v$  such that  $F - \{v\}$  is a non-empty subset of  $S$ .

If  $G = (V(G), E(G))$  is a graph with no independent vertexes, then the dual hyper graph of graph  $G$  (denoted by  $G(A)$ ) is a hyper graph with a vertical range of  $V(G) = E(G)$  and a boundary range of  $E(G) = \{v \in V(G)\}$ , where  $v = \{e \in E(G) \mid v \text{ is the final vertex of } e\}$ . In Chapter 2, we looked at the hyper graph with the definition of edge dominance in the hyper graph. A new definition in the graph called the edge  $H$ -domination is described in chapter 3. An algorithm to find the edge of the  $H$ -dominant graph set is written. It has also been shown that the edge  $H$ -dominating collection obtained by the algorithm is small. The shift in the edge  $H$ -control number of the graph is observed in operations such as removal of the vertex and removal of the edge in this segment. The subset  $T$  of the vertexes in the hypergraph  $G$  is the transverse (also called the vertex cover) in  $G$  where  $T$  has a non-empty intersection with each edge of  $G$ . The transverse number of  $G$  is the minimum transverse scale of  $G$ . The complete cross-section of the hypergraph  $G$  is a cross-section  $T$  with the property that each vertex in  $T$  has at least one neighbor in  $T$  [12]. The complete cross-section is described in graph  $G$ . The complete cross-section of the graph is a vertical cover without separated vertexes. The minimal cardinality of the total cross-section of graph  $G$  is regarded as the total cross-section number (denoted by  $\text{mtt}(G)$ ) of graph  $G$ . The limits on the edge dominant number of the graph are obtained in terms of the



complete transverse number in Chapter 2 and these limits are strengthened with the aid of the edge covering of the graph.

**Definition 1.1:** A set  $S \subseteq V$  is said to be an extended strong vertex cover of the hypergraph  $G$  if the following two conditions hold. (1) If  $\{v\}$  is an edge then  $v \in S$ . (2) If  $u$  and  $v$  are adjacent vertices of  $G$  then  $u \in S$  or  $v \in S$ .

**Proposition 1.2:** Each expanded, solid vertex cover of the hypergraph is a vertex cover Evidence of that. Let  $S$  be an expanded, solid vertex cover and be any edge of the hypergraph  $G$ . If  $e = \{v\}$ , then  $v \in S$ . Therefore  $e$  is true for  $S$  and thus  $e$  is correct for  $S \neq \emptyset$ . If  $e$  includes more than one vertex Enable  $u, v$  to be valid  $e$  with  $e$  to be valid  $S \neq \emptyset$ . So  $u$  and  $v$  are opposite vertices, but  $u \notin S$  and  $v \notin S$ . Well, that's a contradiction. Therefore,  $e$  is true for  $S \neq \emptyset$ . So,  $S$  is the mask of the vertex.

A new definition called a strong edge graph cover is described in chapter 4. The partnership between edge cover and strong edge cover is also explored in this chapter between edge H-domination and strong edge cover.

Concept 1.3. 1.3. Let  $G = (V, E)$  be a hyper-graph. It is said that the  $S$  Set is a secure set if  $S$  does not have an edge  $F$  such as  $|S \cap F| > 1$ . The stability number (denoted by  $\alpha(G)$ ) of the hypergraph  $G$  is defined as the maximum cardinality of the stable set in  $G$ .

Concept 1.4.1.1. Let  $G = (V, E)$  be a hyper-graph. It is stated that the set  $S$  valid  $V$  is an independent set if there are no two vertices of  $S$  adjacent. The maximal cardinality of an isolated set in the hypergraph  $G$  is the independence number (denoted by  $\beta(G)$ ) of the hypergraph  $G$ .

An isolated set of hypergraphs is also regarded as a "strongly stable set." It is clear that the collection of all maximum stable sets in the hypergraph  $G$  includes the collection of all maximum independent sets in  $G$ . It follows that, for any hyper graph  $G$ ,  $\alpha(G) \geq \beta(G)$ . The separate collection of edges shall be considered in Chapter 5. The improvement in the edge independence number is detected as the edge of service is separated from the graph. A new definition called graph edge stability is described in chapter 6. The interaction between edge H-domination and edge stability is explored. The improvement in the edge stability number of the graph is detected during the edge removal process.

Concept 1.5. 1.5. Set  $S$  List  $V$  is claimed to be an expanded isolated set of hyper graphs

$G$  if the following two criteria are fulfilled.



- (1) If the edge is  $\{v\}$ , then the edge is  $v \in S$ .
- (2) If  $u$  and  $v$  are distinct vertices of  $S$  then  $u$  and  $v$  are not adjacent.

### **Proposition 1.6.**

Each expanded independent hypergraph collection is a stable collection. Evidence of that. Let  $S$  be an expanded, stand-alone set of hypergraphs  $G$  and  $S$  includes the edge  $e$ . Suppose that  $e = \{v\}$  then  $v \in S$  is real, which contradicts the concept of an expanded independent collection. Suppose  $e$  includes more than one vertex. Let  $u, v$ , and  $e$  then conclude that  $u, v$ , and  $u$ , are neighboring vertices. This again violates the concept of an expanded independent set.  $S$  must then be a secure package.

A new definition named "solid edge freedom" in graph is described in the chapter.

5. The relationship between the strong edge cover and the strong edge independence of the graph is explored in this part. The improvement in the strong edge independence number is detected as the working edge is separated from the graph. The results of the work are set out in Chapter 7. In this part, we have presented some probable new directions in which this research can be further expanded.

### **OBJECTIVE**

1. Study To Recall that property testing calls for distinguishing objects having a predetermined property from object that are far from any objects that has this property (i.e., are far from the property).
2. Study to Meditating on these facts, one may ask what is the source of this ubiquitous use of graphs in computer science.

### **CONCLUSION**

Graph theory has been the leading alternative in mathematics analysis for many researchers these days. The responsibility for such prominence of science in this region has been attributed to the exponential development and implementation of graph theory in several directions during the last few decades. Graph theory has a strong scope for applications in distinct mathematics, computational computer science, technical sciences, etc. The findings of the study are focused on criteria linked to the edges of the line. Various graph operations, such as removal of vertex, removal of edges and inclusion of edges, are regarded. In fact, the results of these operations on the edge parameters of the graph are observed. In Chapter 2, the edge dominance of the graph is



regarded and the results of the elimination of the vertex, the displacement of the edge and the inclusion of the edge operations on the edge domination number are noted. There is also a possibility to find the required and adequate condition when applying the removal of the vertex graph operations, the removal / addition of the edge on the line, and the domain number of the edge does not alter.

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